TIME-SPACE REGULARIZATION OF THE INVERSE PROBLEM OF ELECTROCARDIOGRAPHY

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Abstract

This paper presents a new method for the regularization of the inverse problem of electrocardiography. Our goal is to reconstruct the epicardial potentials (EPs) from measured body surface potentials. This non-invasive technique is useful for the diagnostic of the cardio-vascular diseases. In our approach, the time-correlation between EPs is used as the regularizing *a priori* information. This information is introduced via the state-space representation proposed in [1]. Our contribution is the derivation of maximum likelihood estimators for identification of the parameters of the state-space model, and in a second stage, for determination of the EPs. Therefore, all unknown quantities are determined from the only available data: the body surface potentials.

1 Problem statement

The goal of the inverse problem of electrocardiography is to quantitatively evaluate the cardiac electrical activity from measured torso potentials. The relationship between body surface potentials and EPs that can be considered linear with a good approximation. Therefore, at time-sample n, one can write

$$\Phi_T(n) = H\Phi_E(n) + \omega(n) \quad n = 1\ldots N, \tag{1}$$

where Φ_T and Φ_E respectively denote the vectors of body surface potentials and EPs, and where H is the transfer matrix between the heart and the torso; ω is a zero-mean Gaussian white noise vector with covariance matrix rI(I is the identity matrix) which represents modeling and measurement errors. In this study, H was computed using a 3-D finite element model of the torso according to the technique described in [2].

It is well-known that this problem is ill-posed, and that it must be regularized through incorporation of prior information on the solution. In the classical regularization methods (Tikhonov regularization or regularization-truncation) [3], the time-correlation of the cardiac activation process is not accounted for as the data are independently processed time-frame by time-frame.

Oster & Rudy [4] pointed out the importance of temporal information for regularization, but they did not explain how to account for this information in a non-invasive manner. More recently, Brooks & al. [5] proposed a partial answer, but their formulation of the *a priori* information lacks flexibility and the estimation of the regularization parameters remains a problem.

In order to account for the time-dependence across time-frames, Joly & al. [1] proposed the following time-invariant linear prediction equation:

$$\Phi_E(n) = F\Phi_E(n-1) + v(n) \quad n = 1\ldots N, \tag{2}$$

where F denotes a time-invariant transition matrix and v a zero-mean Gaussian white vector with covariance matrix qI. Experiments performed for several patients showed the validity of the above model, but also indicated that the model parameters vary greatly from patient to patient. Therefore, model parameters $\{F, q, r\}$ must be specifically estimated for each patient. Here, we propose a method for solving this problem and reconstructing the EPs using torso potentials only.

2 Method

We start with estimation of the EPs when model (2) is known. With our maximum likelihood approach, the estimate $\hat{\Phi}_E(n)$ maximizes Gaussian density $p(\Phi_E \mid \Phi_T; F, q, r)$, or, equivalently, minimizes the following least-squares criterion:

$$\begin{aligned} J &= \sum_{n=1}^{N} \| \Phi_T(n) - H\Phi_E(n) \|^2 \\ &\quad + \lambda \| \Phi_E(n) - F\Phi_E(n-1) \|^2, \end{aligned} \tag{3}$$

where $\lambda = r/q$. Easy computation of $\hat{\Phi}_E(n)$ can be carried out with a Kalman smoother.

We now turn to maximum likelihood estimation of $\{F, q, r\}$ from torso potentials only. The likelihood function is defined by $p(\Phi_T(1), \cdots, \Phi_T(N) \mid F, q, r)$. Under our Gaussian assumption, the likelihood can be computed recursively for any value of $\{F, q, r\}$, by using Kalman

filtering equations. However, maximization of the likelihood presents difficulties, as no closed-form solution is available. Here, we propose to maximize the likelihood with an expectation-maximization (EM) algorithm. The EM algorithm is an iterative procedure that guarantees an increase of the likelihood at each iteration and that converges to a stationary point of the likelihood. The EM algorithm is well suited to our problem because it can be implemented using by-products of the same Kalman smoother as the one used for computation of $\hat{\Phi}_E(n)$ [6], and because it allows for various parametric forms of matrix F. This last point is important in practice because the number of observed data is rather limited compared to the number of elements in F. Therefore, parameterization of F is necessary so as to improve the conditioning of the identification problem.

3 Results and conclusion

EPs were measured with a 63-lead mapping system using a sock electrode array in 8 patients with Wolff-Parkinson-White syndrome who had undergone arrhythmia surgery. Each recording was taken during normal sinus rhythm and lasted 1.024 s. Body surface potentials were simulated by multiplying EPs with transfer matrix H and adding Gaussian noise to the results with a signal-to-noise ratio of 20 dB.

In a first stage, several parametric forms of F were estimated from the EPs of each patient. Then, the EPs were reconstructed using the estimated model and the simulated torso potentials using Kalman smoothing (supervised method). Not surprisingly, we observed that EP reconstructions are better as the number parameters of F grows up.

In second stage, $\{F, q, r\}$ and the EPs were estimated for each patient from the simulated torso potentials using the method described above. Matrix F was parameterized as:

$$F = \alpha I + \beta S. \tag{4}$$

α and β are two unknown scalars parameters; matrix S, known in advance, represents the time-space correlations between EPs at any given lead and their four closest neighbors at the previous time-sample. This structure relies on the assumption that propagation of the EPs is a local phenomenon.

Our results indicate that the EM algorithm yields satisfactory estimates of $\{F, q, r\}$ when constraint (4) is used. Corresponding EP estimates exhibit a lower reconstruction error and a better localization of the extrema than Tikhonov-regularized solutions (see figure 1). Therefore, the proposed method is able to account for time-space correlations in an effective manner, and to improve the solution of the inverse problem of electrocardiography.

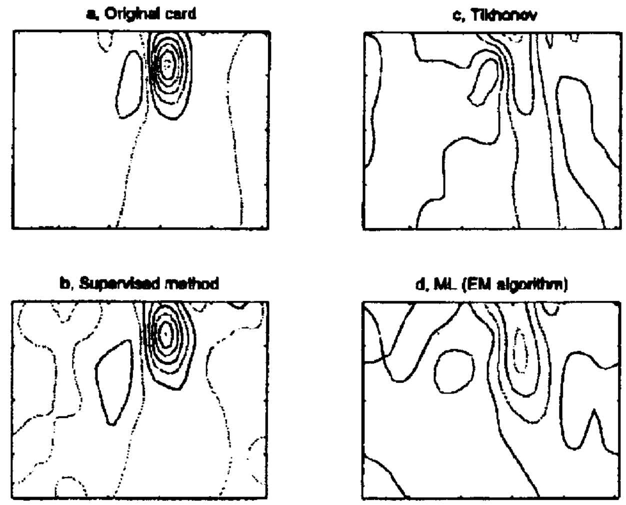

Figure 1: Reconstructed EP cards. (a) Actual EPs. (b) Reconstructed EPs using the supervised method. (c) Reconstructed EPs using Tikhonov regularization. (d) Reconstructed EPs using the proposed method. Our method yields better localization of the extrema and a lower reconstruction error than Tikhonov regularization.

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